Week 4: Heisenberg Uncertainty Principle, EPR Paradox

COMS 4281 (Fall 2024)

Admin

1. Practice problem sheet available, quiz on Gradescope tonight. Quizzes should be done individually.
2. Pset1 out (finally!), due October 6, 11:59pm.
3. Use qBraid instead of Google Colab.
4. Use EdStem to find pset collaborators. **However you must write your own solutions**.

Upcoming Events

- **Seminar**: Quantum Science with Tweezer Arrays.
  - When, where: Monday, Sept 23, 12:30pm, Pupin 8th floor.
  - Who: Manuel Endres (Caltech).

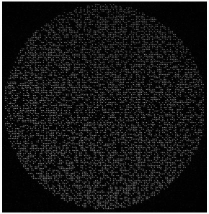

Figure 1: 6000 cesium atoms

Recap

- Partial measurements
- Quantum teleportation
- Non-standard measurements

Heisenberg Uncertainty Principle

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Popular science version: can't exactly know both the position and momentum of a particle simultaneously.

Heisenberg Uncertainty Principle

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In other words, if measuring $|\psi\rangle$ in standard basis yields a deterministic outcome, then it **cannot** have a deterministic outcome if measured according to diagonal basis.

Heisenberg Uncertainty Principle

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It's reasoning about **counterfactual scenarios**: measuring $|\psi\rangle$ in the standard basis, **or** measuring $|\psi\rangle$ in the diagonal basis.

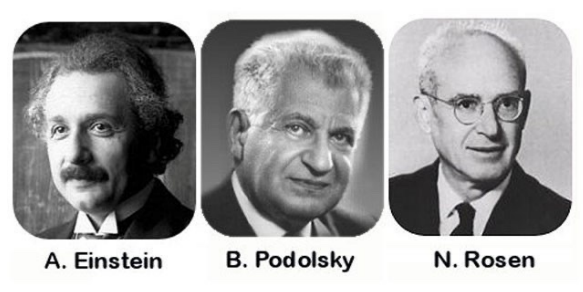
Heisenberg Uncertainty Principle

We say that the standard basis and diagonal basis are **incompatible** or **complementary**.

In quantum physics, the position and momentum of a particle correspond to incompatible measurements!

EPR Paradox

The EPR Paradox



In 1935, Einstein, Podolsky, and Rosen published a paper called

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

The EPR Paradox

The EPR thesis:

Quantum mechanics may be very good at predicting outcomes of experiments, but it cannot be a **complete** *description of Nature.*

The reason they thought this was because of a thought experiment.

The EPR Paradox

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Consider two possible experiments:

- **Experiment A**: Alice measures her qubit in the standard basis $\{|0\rangle, |1\rangle\}$
- **Experiment B**: Alice measures her qubit in the diagonal basis $\{|+\rangle, |-\rangle\}$

Experiment A

Alice gets outcome

- $|0\rangle$ with probability $1/2$, and the post-measurement state is $|00\rangle$.
- $|1\rangle$ with probability $1/2$, and the post-measurement state is $|11\rangle$.

Experiment B

To calculate the probability of getting outcome $|+\rangle$, we use the partial measurement + nonstandard basis rules: first, compute the vector

$$|v_+\rangle = \Big(\langle +| \otimes I\Big)\ |\Phi\rangle = \frac{1}{\sqrt{2}}\Big(\langle + \mid 0\rangle\ |0\rangle\ +\ \langle + \mid 1\rangle\ |1\rangle\Big)$$

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$$= \frac{1}{\sqrt{2}}\Big(\frac{1}{\sqrt{2}} \, |0\rangle + \frac{1}{\sqrt{2}} \, |1\rangle \Big) = \frac{1}{2} \, |0\rangle + \frac{1}{2} \, |1\rangle \, .$$

The squared length is $\| \, |v_+\rangle \, \|^2 = \frac{1}{2^2} + \frac{1}{2^2} = \frac{1}{2}$.

Experiment B

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Similarly, the probability of getting outcome $|-\rangle$ is $\frac{1}{2}$ and the post-measurement state is $|-\rangle \otimes |-\rangle$.

The EPR Paradox

Alice and Bob are in far-away galaxies and share the EPR state $|\Phi\rangle = \frac{1}{\sqrt{2}}\Big(|00\rangle + |11\rangle \Big)$. If Alice measures in standard basis, after seeing her result she knows exactly what state Bob's qubit is in – even if Bob's qubit is zillions of lightyears away.

The EPR Paradox

Einstein's (and Podolsky's and Rosen's) reasoning:

1. If Alice did Experiment A and got outcome (say) $|0\rangle$, then it must have been Bob's qubit was really $|0\rangle$ all along.

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3. But Alice's choice of measurement (standard or diagonal) couldn't have made an instantaneous difference in intrinsic the state of Bob's qubit, right?

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3. But Alice's choice of measurement (standard or diagonal) couldn't have made an instantaneous difference in intrinsic the state of Bob's qubit, right?
4. Therefore Bob's qubit must have answers prepared for both Experiments simultaneously – violating Heisenberg's Uncertainty Principle!

The EPR Paradox

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EPR's thesis: There is a **deeper** classical theory – called a **local hiden variable theory** – that

- Reproduces the same statistics as Quantum Mechanics
- But has hidden variables that describes the intrinsic state of particles.
- Respects the speed of light limit.

The EPR Paradox

What do you think of Einstein's reasoning?

It is a paradox?