COMS 4281 - Introduction to Quantum Computing Fall 2025

Practice Worksheet 1 - Reversible Computing and Quantum Math

Problem 1: Reversible transformations

In class we covered how to take any boolean function $f : \{0,1\}^n \to \{0,1\}$ and convert it to its reversible equivalent R_f. Here we will convert a 2-bit function. Let $f : \{0,1\}^2 \to \{0,1\}$ be given by the truth table below.

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	0
1	0	0
1	1	1

So $f(x_1, x_2) = 1$ when the two input bit are equal otherwise known as ¬XOR and define the reversible function (3-bit) by $R_f : (x_1, x_2, b) \mapsto (x_1, x_2, b \oplus f(x_1, x_2))$.

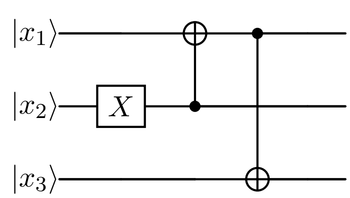
(a) Fill in the truth table of R_f for each 3-bit input (x_1, x_2, b) then write the corresponding output.

(b) Explain in a sentence why the mapping must be a permutation of $\{0,1\}^3$.

(c) What is the inverse transformation of R_f?

Problem 2: 3-bit Circuit to Truth Table

Below is a reversible circuit comprised of $CNOT$ and X gates. It defines a reversible transformation on 3 bits. Write out the truth table of the transformation (i.e., what does every 3 bit input map to?).



Problem 3: Truth table to 2-bit circuit

Let $F : \{0,1\}^2 \mapsto \{0,1\}^2$ be the reversible function given by the truth table below.

x_1	x_2	x_1'	x_2'
0	0	1	1
0	1	0	1
1	0	1	0
1	1	0	0

Construct the corresponding 2-bit reversible circuit using $CNOT$ and X gates that implement F.

Problem 4: Matrix notation

Recall our convention for mapping the states $|0\rangle, |1\rangle$ of a bit to column vectors:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

In class we discussed how we use the tensor product notation to describe the state of a composite system (e.g. a system consisting of multiple bits). For example, $|0\rangle \otimes |1\rangle$ describes the state of two bits where the first is in state $|0\rangle$, and the second is in state $|1\rangle$. When we translate to vector notation, however, we have to adhere to a convention. There are many possible conventions, but the one we will use throughout class is the one where $|0\rangle \otimes |0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $|0\rangle \otimes |1\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$, and so on. Furthermore we use the following abbreviations:

$$|a\rangle \otimes |b\rangle = |a\rangle |b\rangle = |a, b\rangle = |ab\rangle \ .$$

Let

$$R = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \ .$$

(a) What is the 4-dimensional column vector corresponding to $R|1,0\rangle$?

(b) What is the ket state corresponding to $R|1,0\rangle$?

(c) What reversible circuit does R correspond to?