

No-Cloning, Teleportation, Measurements, and Uncertainty

COMS 4281 (Fall 2026)

October 9, 2026

No-Cloning Theorem

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No-Cloning Theorem: Quantum Info cannot be copied.

Formal statement: there is no two-qubit unitary U such that for all single-qubit states $|\psi\rangle \in \mathbb{C}^2$,

$$U |\psi\rangle \otimes \underbrace{|0\rangle}_{\text{ancilla}} = |\psi\rangle \otimes |\psi\rangle$$

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but unitary operators have to preserve inner products!

No-Cloning Theorem Proof

So that means

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Contradiction!

Quantum Teleportation

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She could try to call Bob over the phone and send a classical description of $|\psi\rangle$, but that could require an infinite number of bits if $|\psi\rangle$ has amplitudes that use transcendental numbers.

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Suppose one year ago, Alice and Bob were in the same quantum lab, and they generated the entangled state

$$|\text{EPR}\rangle = \frac{1}{\sqrt{2}} \left(|00\rangle + |11\rangle \right) .$$

Alice takes the first qubit, Bob takes the second qubit and they go their separate ways.

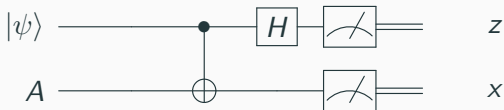
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She performs the following circuit on her two qubits:



where A denotes Alice's half of the EPR pair, and the measurement outcomes are denoted $z, x \in \{0, 1\}$ respectively.

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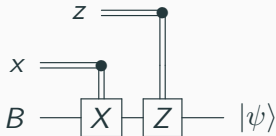
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Bob: “Nice! What corrections do I need to do?”

Alice tells Bob the bits z, x .

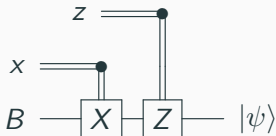
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Bob then applies the following gates to his EPR qubit B , depending on the values of x, z .



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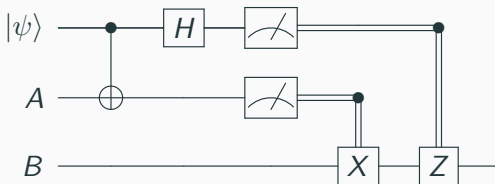


Recall that

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

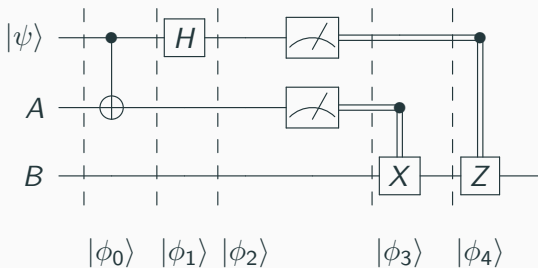
Quantum teleportation

Viewing the entire process as a circuit, this looks like



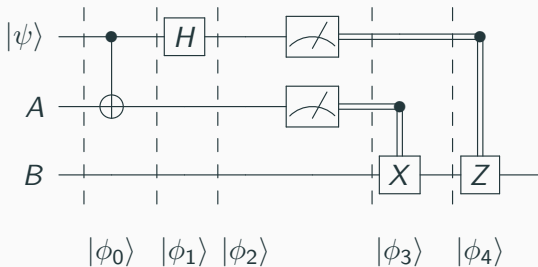
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We analyze it step by step:



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Let's do this together on the board!

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Both the preshared entanglement, and the classical communication are necessary for quantum teleportation to work.

Entanglement does not allow for faster-than-light communication!

Nonstandard Measurements

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This is what we call a **standard basis measurement** or a **computational basis measurement**: the outcomes are the standard basis (a.k.a. computational basis) vectors

$$\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \dots \quad \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

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Measuring in diagonal basis

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Rewrite $|\psi\rangle$ as a linear combination of the $\{|+\rangle, |-\rangle\}$ basis:

$$|\psi\rangle = \alpha \frac{\sqrt{2}}{2} (|+\rangle + |-\rangle) + \beta \frac{\sqrt{2}}{2} (|+\rangle - |-\rangle)$$

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Measuring in diagonal basis

The probability of getting $|+\rangle$ outcome is thus

$$\left(\frac{\sqrt{2}}{2}\right)^2 \cdot |\alpha + \beta|^2$$

and similarly the probability of $|-\rangle$ outcome is

$$\left(\frac{\sqrt{2}}{2}\right)^2 \cdot |\alpha - \beta|^2$$

Another perspective

Geometric perspective: measuring $|\psi\rangle$ in diagonal basis is **projecting** $|\psi\rangle$ along the $|+\rangle, |-\rangle$ axes. The probability of either outcome is the square of the magnitude of the projections.

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Algebraically, we apply the projection matrices $|+\rangle\langle+|$ and $|-\rangle\langle-|$ to $|\psi\rangle$. Projection of $|\psi\rangle$ along $|+\rangle$ is

$$|v\rangle = |+\rangle\langle+|\psi\rangle = \langle+|\psi\rangle |+\rangle .$$

The probability of getting $|+\rangle$ outcome is equal to

$$\| |v\rangle \|^2 = \| \langle+|\psi\rangle |+\rangle \|^2 = |\langle+|\psi\rangle|^2 \cdot \| |+\rangle \|^2 = |\langle+|\psi\rangle|^2$$

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Similarly probability of getting $|-\rangle$ outcome is $|\langle-|\psi\rangle|^2$.

General formula for measuring in a basis

Let $|\psi\rangle \in \mathbb{C}^d$ be a quantum state. Let $B = \{|b_1\rangle, \dots, |b_d\rangle\}$ be an **orthonormal basis** for $\mathbb{C}^d$.

Measuring $|\psi\rangle$ with respect to basis B yields outcome $|b_j\rangle$ with probability

$$\left| \langle b_j | \psi \rangle \right|^2$$

and the post-measurement state is $|b_j\rangle$.

Implementing measurements in different bases

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Measuring $|\psi\rangle$ in a basis B is **equivalent** to

1. Apply a unitary V that maps B to standard basis (i.e. $|b_j\rangle \rightarrow |j\rangle$).
2. Measure $V|\psi\rangle$ in standard basis to get outcome $|j\rangle$.
3. Apply inverse unitary $V^\dagger$ to map $|j\rangle$ back to $|b_j\rangle$ as the post-measurement state.

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After it collapses to $|0\rangle$, we apply $H^\dagger = H$ to get $|+\rangle$, the post-measurement state.

Partial measurements plus nonstandard measurements

Let's combine the two concepts! Let $|\psi\rangle = \sum_{i,j} \alpha_{ij} |i\rangle \otimes |j\rangle$ denote a two-qubit state. Say we measure the first qubit with respect to basis $\{|b_0\rangle, |b_1\rangle\}$.

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Measurement rule: Equivalent to applying the projections

$$\begin{aligned} |v_0\rangle &= \left(|b_0\rangle \langle b_0| \otimes I \right) |\psi\rangle \\ |v_1\rangle &= \left(|b_1\rangle \langle b_1| \otimes I \right) |\psi\rangle . \end{aligned}$$

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The probability of getting $|b_0\rangle$ outcome is $\| |v_0\rangle \|^2$, and the post-measurement state is $|v_0\rangle / \| |v_0\rangle \|$.

Similarly, probability of getting $|b_1\rangle$ outcome is $\| |v_1\rangle \|^2$, and post-measurement state is $|v_1\rangle / \| |v_1\rangle \|$.

Projecting $|\psi\rangle$ along the first projection yields

$$\begin{aligned} \left(|b_0\rangle \langle b_0| \otimes I \right) |\psi\rangle &= \sum_{i,j} \alpha_{ij} |b_0\rangle \langle b_0|i\rangle \otimes I |j\rangle \\ &= \sum_{i,j} \alpha_{ij} \langle b_0|i\rangle |b_0\rangle \otimes |j\rangle \\ &= |b_0\rangle \otimes \left(\sum_{i,j} \alpha_{ij} \langle b_0|i\rangle |j\rangle \right) . \end{aligned}$$

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 &= |b_0\rangle \otimes \left(\sum_{i,j} \alpha_{ij} \langle b_0|i\rangle |j\rangle \right) .
 \end{aligned}$$

The probability of getting $|b_0\rangle$ outcome is

$$\begin{aligned}
 \left\| |b_0\rangle \otimes \left(\sum_{i,j} \alpha_{ij} \langle b_0|i\rangle |j\rangle \right) \right\|^2 &= \underbrace{\| |b_0\rangle \|^2}_1 \cdot \left\| \sum_{i,j} \alpha_{ij} \langle b_0|i\rangle |j\rangle \right\|^2 \\
 &= \left\| \sum_j \left(\sum_i \alpha_{ij} \langle b_0|i\rangle \right) |j\rangle \right\|^2 \\
 &= \sum_j \left| \sum_i \alpha_{ij} \langle b_0|i\rangle \right|^2 .
 \end{aligned}$$

Partial measurements plus nonstandard measurements

Conditioned on the outcome $|b_0\rangle$ in the first qubit, the post-measurement state of the two qubits is then

$$|b_0\rangle \otimes \frac{\sum_{i,j} \alpha_{ij} \langle b_0|i\rangle |j\rangle}{\left\| \sum_{i,j} \alpha_{ij} \langle b_0|i\rangle |j\rangle \right\|} = |b_0\rangle \otimes \frac{\sum_{i,j} \alpha_{ij} \langle b_0|i\rangle |j\rangle}{\sqrt{\sum_j \left| \sum_i \alpha_{ij} \langle b_0|i\rangle \right|^2}}$$

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Notice that, even though the original state $|\psi\rangle$ could have been entangled, the post-measurement state is **unentangled**!

Measurement generally destroy entanglement.

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Let's work through an example:

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle.$$

Measure the **first** qubit in the **diagonal basis**.

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Measure the **first** qubit in the **diagonal basis**.

- What is the probability the outcome is $|+\rangle$? Or $|-\rangle$?
- What is the post-measurement state in either case?

Heisenberg Uncertainty Principle

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Popular science version: can't exactly know both the position and momentum of a particle simultaneously.



Heisenberg Uncertainty Principle

In quantum information theory terms: it is not possible for a qubit $|\psi\rangle \in \mathbb{C}^2$ to be simultaneously determined in both the **standard** basis and the **diagonal basis**.

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In quantum information theory terms: it is not possible for a qubit $|\psi\rangle \in \mathbb{C}^2$ to be simultaneously determined in both the **standard** basis and the **diagonal basis**.

In other words, if measuring $|\psi\rangle$ in standard basis yields a deterministic outcome, then it **cannot** have a deterministic outcome if measured according to diagonal basis.

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Quantitatively, let:

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Then a version of Heisenberg's uncertainty principle says

$$(p_0 - p_1)^2 + (p_+ - p_-)^2 \leq 1$$

.

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Important point: it's not about what happens if you **sequentially** measure the state $|\psi\rangle$ (what happens then?).

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It's reasoning about **counterfactual scenarios**: measuring $|\psi\rangle$ in the standard basis, **or** measuring $|\psi\rangle$ in the diagonal basis.

Heisenberg Uncertainty Principle

We say that the standard basis and diagonal basis are **incompatible** or **complementary**.

In quantum physics, the position and momentum of a particle correspond to incompatible measurements!